

Non-universal shear viscosity from Einstein gravity

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Abstract

A very famous result of gauge/gravity duality is the universality of the ratio of shear viscosity to entropy density in every field theory holographically dual to classical, two-derivative (Einstein) gravity. We present a way to obtain deviation from this universality by breaking the rotational symmetry spontaneously. In anisotropic fluids additional shear modes exist and their corresponding shear viscosities may be non-universal. We confirm this by explicitly calculating the shear viscosities in a transversely isotropic background, a p-wave superfluid, and study its critical behavior. This is a first decisive step towards further applications of gauge/gravity duality to physical systems.

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Gauge/gravity duality [1] provides a novel method for studying strongly-coupled systems at finite temperature and densities. As such it is expected to have useful applications to the quark-gluon plasma as well as to condensed matter physics. The duality also allows to calculate physical observables in the real-time formalism, e. g. retarded correlation functions [2, 3] and hydrodynamic transport coefficients [4]. The most important result of these studies is the universality of the ratio of the shear viscosity η to the entropy density s in every field theory holographically dual to classical, two-derivative (Einstein) gravity theory [5, 6, 7, 8]. The universal number $\eta/s = 1/4\pi$ (in natural units) fits surprisingly well the measured quantities in strongly-coupled real-world systems such as the quark-gluon plasma and cold fermions at unitarity, see e. g. [9]. The classical, two-derivative gravity theory is mapped to a gauge theory with large rank of the gauge group (large N_c) and to a large 't Hooft coupling λ .

Since the ratio of shear viscosity to entropy density usually depends on the temperature and is non-universal in real-world systems, it is important to find deviations from the universal behavior in order to make further contact with real-world systems. In this letter, we present a deviation from the universal

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ratio at leading order in N_c and λ . So far a deviation from the universality only occurs in subleading corrections of N_c and/or λ , see e. g. [10, 11].

The universality of η/s strongly depends on the shear mode transforming as a helicity two state under the rotational symmetry. Since it is the only helicity two state in the system, the shear mode decouples from the other modes and it can effectively be written as a minimally coupled scalar. The universality of η/s is then related to the universality of the coupling of this minimally coupled scalar. The coupling is always given by the gravitational coupling constant since the shear mode is a graviton [8]. We circumvent this important assumption that the shear mode is a helicity two state by breaking the rotational symmetry spontaneously. A fluid with spontaneously broken rotational symmetry is anisotropic and more than one shear mode exists. Since some of these modes do not transform as helicity two states, the corresponding shear viscosities can be non-universal [12, 13].

The viscosity, including shear and bulk viscosity, in anisotropic fluids is described by a rank four tensor with in general 21 independent coefficients [14]. In the following we study systems with p-wave symmetry (transversely isotropic). This reduces the independent coefficients in the viscosity tensor to five. Two of these independent coefficients are shear viscosities [15]. We explicitly calculate these two shear viscosities by using the recipe for the holographic calculation of the retarded correlators [2, 3] and Kubo formulae [16] which are given by

$$\eta_i = - \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} G_{i,i}^R(\omega, 0) \quad \text{with} \quad i \in \{xy, xz, yz\}, \quad (1)$$

where the x -axis is the preferred direction and $\eta_{xy}(= \eta_{xz})$, η_{yz} are the two independent shear viscosities. The retarded correlation function of the energy-momentum tensor $T_{\mu\nu}$ is defined by

$$G_{ij,kl}^R(\omega, 0) = -i \int dt dx e^{i\omega t} \theta(t) \langle [T_{ij}(t, x), T_{kl}(0, 0)] \rangle. \quad (2)$$

We present more details on anisotropic shear viscosities in the appendix below.

In the holographic context, the spontaneous breaking of symmetries by black holes developing hair was first achieved in [17] and later used to construct holographic superconductors/superfluids by breaking an abelian symmetry [18]. Along this line also p-wave superconductors/superfluids have been constructed [19] and gave rise to the first string theory embeddings of holographic superconductors/superfluids [20, 21, 22]. In p-wave superconductors/superfluids, also the spatial rotational symmetry is spontaneously broken in addition to the internal abelian symmetry. In order to obtain the effects of spontaneous rotational symmetry breaking for the energy-momentum tensor, which determines the hydrodynamics of the systems, we have to take the back-reaction of the superfluid into account. This was obtained e. g. in [23]. On the gravity side, the p-wave superfluid state corresponds to an asymptotically AdS black hole which carries vector hair.

In order to construct p-wave superfluid states, we consider $SU(2)$ Einstein-Yang-Mills theory in $(4 + 1)$ -dimensional asymptotically AdS space as in [23].

The action is

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[(R - \Lambda) - \frac{\alpha^2}{2} F_{\mu\nu}^a F^{a\mu\nu} \right], \quad (3)$$

where κ_5 is the five-dimensional gravitational constant, $\Lambda = -\frac{12}{L^2}$ is the cosmological constant, with L being the AdS radius, and $\alpha = \kappa_5/\hat{g}$ the ratio of the gravitational constant κ_5 to the Yang-Mills coupling constant \hat{g} . The $SU(2)$ field strength $F_{\mu\nu}^a$ is

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c, \quad (4)$$

where $\mu, \nu = \{t, r, x, y, z\}$, with r being the AdS radial coordinate, and ϵ^{abc} is the totally antisymmetric tensor with $\epsilon^{123} = +1$. The A_μ^a are the components of the matrix-valued gauge field, $A = A_\mu^a \tau^a dx^\mu$, where the τ^a are the $SU(2)$ generators, which are related to the Pauli matrices by $\tau^a = \sigma^a/2i$.

Following ref. [19], to construct charged black hole solutions with vector hair we choose a gauge field ansatz

$$A = \phi(r) \tau^3 dt + w(r) \tau^1 dx. \quad (5)$$

The motivation for this ansatz is as follows. In the field theory we introduce a chemical potential for the $U(1)$ symmetry generated by τ^3 . We denote this $U(1)$ as $U(1)_3$. The gravity field dual to the $U(1)_3$ density is A_t^3 , hence we include $A_t^3(r) \equiv \phi(r)$ in our ansatz. We allow for states with a nonzero $\langle J_1^x \rangle$, so in addition we introduce $A_x^1(r) \equiv w(r)$. With this ansatz for the gauge field, the Yang-Mills stress-energy tensor is diagonal. Solutions with nonzero $w(r)$ preserve only an $SO(2)$ subgroup of the $SO(3)$ rotational symmetry, so our metric ansatz respects only $SO(2)$. Furthermore, given that the Yang-Mills stress-energy tensor is diagonal, a diagonal metric is consistent. Our metric ansatz is

$$ds^2 = -N(r) \sigma(r)^2 dt^2 + \frac{1}{N(r)} dr^2 + r^2 f(r)^{-4} dx^2 + r^2 f(r)^2 (dy^2 + dz^2), \quad (6)$$

with $N(r) = -\frac{2m(r)}{r^2} + \frac{r^2}{L^2}$. For our black hole solutions we denote the position of the horizon as r_h . The AdS boundary is at $r \rightarrow \infty$.

Inserting our ansatz into the Einstein and Yang-Mills equations yields five equations of motion for $m(r)$, $\sigma(r)$, $f(r)$, $\phi(r)$, $w(r)$ and one constraint equation from the rr component of the Einstein equations. The dynamical equations can be found in [23]. Using scale transformations, we can set the boundary values of both $\sigma(r)$ and $f(r)$ to one, so that the metric will be asymptotically AdS.

A known analytic solution of the equations of motion is an asymptotically AdS Reissner-Nordström black hole, which has $\phi(r) = \mu - q/r^2$, $w(r) = 0$, $\sigma(r) = f(r) = 1$, and $N(r) = \left(r^2 - \frac{2m_0}{r^2} + \frac{2\alpha^2 q^2}{3r^4} \right)$, where $m_0 = \frac{r_h^4}{2} + \frac{\alpha^2 q^2}{3r_h^2}$ and $q = \mu r_h^2$. Here μ is the value of $\phi(r)$ at the boundary, which in CFT terms is the

$U(1)_3$ chemical potential. Since $w = 0$, i. e. $\langle J_1^x \rangle = 0$, this solution corresponds to the normal phase of the system. To find solutions with nonzero $w(r)$ we resort to numerics. We solve the equations of motion using a shooting method (see [23] for details).

We calculate the shear viscosity in this background via the Kubo formulae by considering two point functions of the energy-momentum tensor and the currents¹. As described in [2, 3, 24], these two point functions are determined by considering fluctuations of the metric $h_{\mu\nu}$ and the gauge fields a_μ^a about this background. For these fluctuations we choose the ansatz

$$\begin{aligned} h_{\mu\nu}(t, \vec{x}, r) &= h_{\mu\nu}(r) e^{-i\omega t + i\vec{k} \cdot \vec{x}}, \\ a_\mu^a(t, \vec{x}, r) &= a_\mu^a(r) e^{-i\omega t + i\vec{k} \cdot \vec{x}}. \end{aligned} \quad (7)$$

Taking the gauge freedom into account, we may set the modes a_r^a and $h_{\mu r}$ for $\mu \in \{t, \vec{x}, r\}$ to zero. This leads to eight constraints in addition to the equations of motion for the 22 dynamical fields. This reduces the number of the physical modes of the system to 14.

The shear viscosities are determined by the zero momentum correlation functions. Thus we only consider time dependent fluctuations, i. e. $\vec{k} = 0$. These fluctuations can be characterized by their transformation properties under the unbroken $SO(2)$ rotational symmetry:

$$\begin{aligned} \text{helicity 2 : } & h_{yz}, h_{yy} - h_{zz} \\ \text{helicity 1 : } & h_{yt}, h_{xy}, h_{yr}; a_y^a \\ & h_{zy}, h_{xz}, h_{zr}; a_z^a \\ \text{helicity 0 : } & h_{tt}, h_{yy} + h_{zz}, h_{xx}, h_{xt}, h_{xr}, h_{tr}, h_{rr}; \\ & a_t^a, a_x^a, a_r^a. \end{aligned} \quad (8)$$

The 14 physical modes decouple in several blocks. The first block contains the usual two physical helicity two modes including the shear mode h_{yz} . In the helicity one block there are eight physical modes which can be split in two blocks by the residual $SO(2)$ rotational symmetry. In this block the additional shear modes h_{xy} and h_{xz} appear. The helicity zero block contains additional four physical modes.

Since the first shear mode h_{yz} is the usual helicity two mode, the general proofs of universality [5, 6, 7, 8] apply and we obtain $\eta_{yz}/s = 1/4\pi$. The second shear mode h_{xy} is a helicity one mode and couples to other physical helicity one modes. Thus the assumptions of the proofs of universality [5, 6, 7, 8] are not satisfied and the shear viscosity can be non-universal as we will see in the following.

¹The two point functions of the currents have to be considered in addition to the two point functions of the energy-momentum tensor since there are interactions between the energy-momentum tensor and the currents.

Due to the residual \mathbb{Z}_2 symmetries, the four physical helicity one modes decouple into two blocks of coupled differential equations in the case of $\vec{k} = 0$. The first block contains one physical mode which is determined by the dynamical fields h_{yt} and a_y^3 and the constraint $h_{yr} = 0$. This mode determines the electrical [25] and thermal conductivity and the thermoelectric coefficients. The three coefficients are related by Ward identities (see e. g. [26]). The second block contains the three physical modes a_y^1 , a_y^2 , and h_{xy} . We are interested in the second block which contains the shear mode $\Psi = h_{xy}/(r^2 f^2) = h_x^y$. The linearized equations of motion for the modes in the second block are given by

$$\begin{aligned}
0 &= \Psi'' + \left(\frac{1}{r} + \frac{4r}{N} + \frac{6f'}{f} - \frac{r\alpha^2\phi'^2}{3N\sigma^2} \right) \Psi' + \frac{\omega^2\Psi}{N^2\sigma^2} \\
&\quad + \frac{2\alpha^2}{r^2 f^2} \left(w' a_y^{1'} - \frac{w\phi^2 a_y^1}{N^2\sigma^2} + \frac{i\omega w\phi a_y^2}{N^2\sigma^2} \right) \\
0 &= a_y^{1''} + a_y^{1'} \left(\frac{1}{r} - \frac{2f'}{f} + \frac{N'}{N} + \frac{\sigma'}{\sigma} \right) + a_y^1 \left(\frac{\omega^2}{N^2\sigma^2} + \frac{\phi^2}{N^2\sigma^2} \right) - f^6 w' \Psi' \\
&\quad - \frac{2i\omega a_y^2 \phi}{N^2\sigma^2} \\
0 &= a_y^{2''} + a_y^{2'} \left(\frac{1}{r} - \frac{2f'}{f} + \frac{N'}{N} + \frac{\sigma'}{\sigma} \right) + a_y^2 \left(-\frac{f^4 w^2}{r^2 N} + \frac{\omega^2}{N^2\sigma^2} + \frac{\phi^2}{N^2\sigma^2} \right) \\
&\quad + \frac{i\omega\phi}{N^2\sigma^2} (-f^6 w\Psi + 2a_y^1),
\end{aligned} \tag{9}$$

where the prime denotes the derivative with respect to the radial variable r . We solve the coupled differential equations numerically and determine the retarded correlator $G_{xy,xy}^R$ using the recipe of [2, 3, 24],

$$G_{xy,xy}^R(\omega, 0) = + \frac{1}{2\kappa_5^2} r^5 \frac{\Psi'}{\Psi} \Big|_{r \rightarrow \infty} - \text{counter terms}. \tag{10}$$

Using the Kubo formulae (1), we numerically determine the ratio of shear viscosity η_{xy} to entropy density s , which is given by the Bekenstein-Hawking entropy of the black hole $s = 2\pi r_h^3/\kappa_5^2$.

In fig 1 we compare our numerical results for the ratio of the shear viscosity η_{xy} to the entropy density s with the universal behavior of the shear viscosity η_{yz} for different values of the ratio of the gravitational coupling to the Yang-Mills coupling, denoted by α . We see that in the normal phase $T \geq T_c$, the two shear viscosities coincide as required in an isotropic fluid. In addition, the ratio of shear viscosity to entropy density is universal. In the superfluid phase $T < T_c$, the two shear viscosities deviate from each other and η_{xy} is non-universal. However it is exciting that the KSS bound on the ratio of shear viscosity to entropy density [5] is still valid.

The difference between the two viscosities in the superfluid phase is controlled by α . In the probe limit where $\alpha = 0$, the shear viscosities also coincide

in the superfluid phase. By increasing the back-reaction of the gauge fields, i. e. rising α , the deviation between the shear viscosities becomes bigger in the superfluid phase as shown in fig 1. If α is larger than the critical value found in [23] where the phase transition to the superfluid phase becomes first order, the shear viscosities are also multivalued close to the phase transition as seen in fig. 2. Since there is a maximal α denoted by α_{max} for which the superfluid phase exists, we expect that the deviation of the shear viscosity η_{xy} from its universal value is maximal for this α_{max} . Unfortunately numerical calculations for large values of α are very challenging such that we cannot present satisfying numerical data for this region. It is interesting that also the deviations due to λ and N_c corrections are bounded. In this case the bound is determined by causality [27].

For α smaller than the critical value where the phase transition is second order, we may study the critical behavior of the ratio of the shear viscosities to entropy density close to the phase transition. Due to universality, η_{yz}/s is constant and does not change on both sides of the phase transition, while η_{xy}/s is only constant in the normal phase, but has a different critical behavior in the superfluid phase. Let us consider the critical exponent related to η_{xy}/s and its dependence on α . From our numerical data we obtain the critical behavior

$$1 - 4\pi \frac{\eta_{xy}}{s} \propto \left(1 - \frac{T}{T_c}\right)^\beta \quad \text{with} \quad \beta = 1.00 \pm 3\% \quad (11)$$

in the superfluid phase close to the phase transition. It is interesting that the critical exponent β does not change with α . A more precise statement can be made if it is possible to determine this critical exponent analytically. We suggest that this can be achieved by an expansion in the order parameter $\langle J_1^x \rangle$ and in the back-reaction controlled by α . So far, the expansion is known in the probe limit $\alpha = 0$ only [28]. However we expect that this expansion may be extended to small values of α . We plan to study the critical behavior of this system in more detail in the future and to make contact with the theory of dynamical critical phenomena [29].

To our knowledge this is the first non-universal behavior of the shear viscosity to entropy density ratio in a theory which is holographically dual to classical, two derivative Einstein gravity theory. This non-universal behavior is due to the fact that the corresponding gravity fluctuation h_{xy} transforms as a helicity one state and therefore couples to the fluctuations of the gauge field $a_y^{1,2}$. We expect that similar results can be obtained for every gravity background which spontaneously breaks the rotational symmetry.

Appendix A. General Remarks on viscosity in anisotropic fluids

Here we explain why the Kubo formulae (1) are appropriate for calculating the shear viscosities $\eta_{xy}(=\eta_{xz})$ and η_{yz} even in a p-wave superfluid.

In general, viscosity refers to the dissipation of energy due to any internal motion [30]. For an internal motion which describes a general translation or a

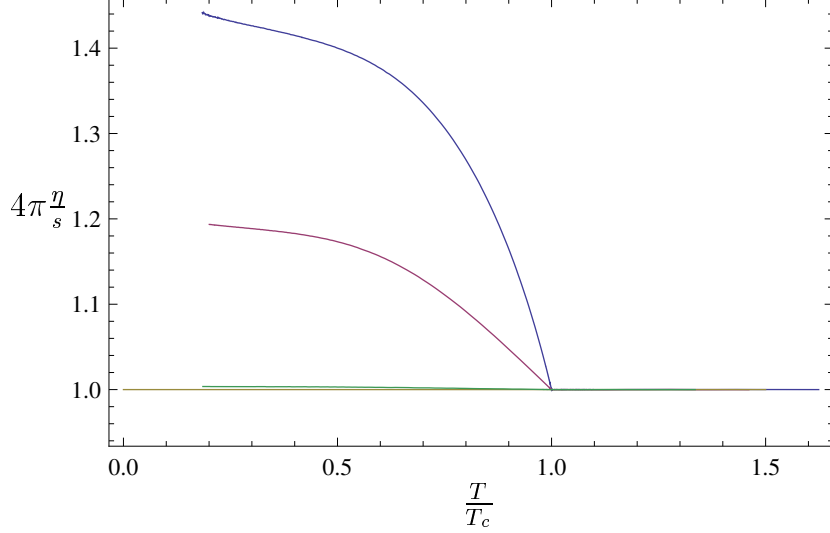


Figure 1: Ratio of shear viscosities η_{yz} and η_{xy} to entropy density s over the reduced temperature T/T_c for different values of the ratio of the gravitational coupling constant to the Yang-Mills coupling constant α : The color coding is as follows: In yellow, η_{yz}/s for all values of α ; while the curve for η_{xy}/s is plotted in green for $\alpha = 0.032$, red for $\alpha = 0.224$ and blue for $\alpha = 0.316$. The shear viscosities coincide and are universal in the normal phase $T \geq T_c$. However in the superfluid phase $T < T_c$, the shear viscosity η_{yz} has the usual universal behavior while the shear viscosity η_{xy} is non-universal.

general rotation, the dissipation is zero. Thus the dissipation depends on the gradient of the velocities u^μ only in the combination $u_{\mu\nu} = \frac{1}{2}(\partial_\mu u_\nu + \partial_\nu u_\mu)$, and we may define a general dissipation function $\Xi = \frac{1}{2}\eta^{\mu\nu\lambda\rho}u_{\mu\nu}u_{\lambda\rho}$, where $\eta^{\mu\nu\lambda\rho}$ defines the viscosity tensor [14]. Its symmetries are given by

$$\eta^{\mu\nu\lambda\rho} = \eta^{\nu\mu\lambda\rho} = \eta^{\mu\nu\rho\lambda} = \eta^{\lambda\rho\mu\nu}. \quad (\text{A.1})$$

The part of the stress tensor which is dissipative due to viscosity is defined by

$$T_{\text{diss}}^{\mu\nu} = -\frac{\partial\Xi}{\partial u_{\mu\nu}} = -\eta^{\mu\nu\lambda\rho}u_{\lambda\rho}. \quad (\text{A.2})$$

We consider a fluid in the rest frame of the normal fluid $u^t = 1$. To satisfy the condition of the Landau frame $u_\mu T_{\text{diss}}^{\mu\nu} = 0$, the stress-energy tensor and thus the viscosity has non-zero components only in the spatial directions $i, j = \{x, y, z\}$. In general only 21 independent components of η_{ijkl} appear in the expressions above.

For an isotropic fluid, there are only two independent components which are usually parameterized by the shear viscosity η and the bulk viscosity ζ . The dissipative part of the stress tensor becomes $T_{ij}^{\text{diss}} = -2\eta(u_{ij} - \frac{1}{3}\delta_{ij}u_l^l) - \zeta u_l^l \delta_{ij}$ which is the well-known result.

In a transversely isotropic fluid, there are five independent components of the tensor η_{ijkl} . For concreteness we choose the symmetry axis to be along the

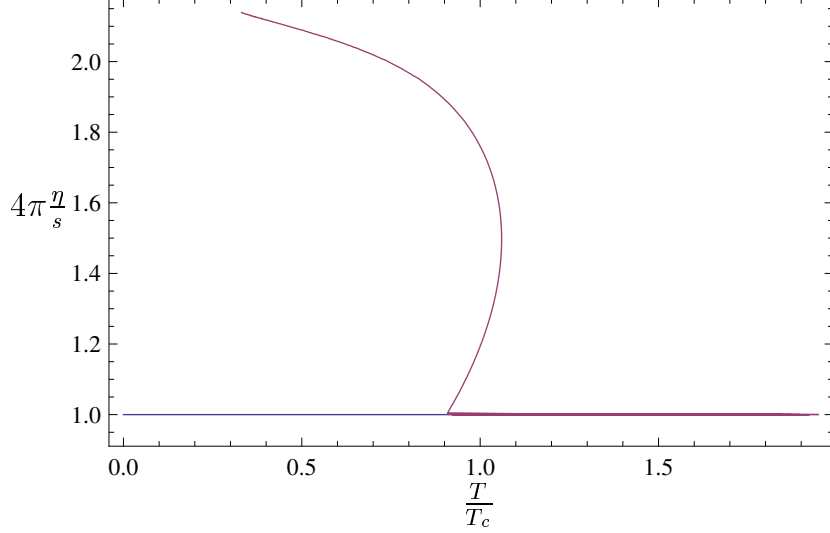


Figure 2: Ratio of shear viscosities η_{yz} (blue) and η_{xy} (red) to entropy density s over the reduced temperature T/T_c for $\alpha = 0.447$, which is bigger than the critical value where the phase transition becomes first order: The shear viscosities coincide in the normal phase $T \geq T_c$ and are universal. In the superfluid phase η_{xy} is non-universal. Close to the phase transition, it is multivalued as expected for a first order phase transition.

x -axis. The non-zero components are given by

$$\begin{aligned}
 \eta_{xxxx} &= \zeta_x - 2\lambda, & \eta_{yyyy} &= \eta_{zzzz} = \zeta_y - \frac{\lambda}{2} + \eta_{yz}, \\
 \eta_{xxyy} &= \eta_{xxzz} = \lambda, & \eta_{yyzz} &= \zeta_y - \frac{\lambda}{2} - \eta_{yz}, \\
 \eta_{zyyz} &= \eta_{yz}, & \eta_{xyxy} &= \eta_{xzzx} = \eta_{xy}.
 \end{aligned} \tag{A.3}$$

The non-zero off-diagonal components of the stress tensor are given by

$$\begin{aligned}
 T_{xy}^{\text{diss}} &= -2\eta_{xy}u_{xy}, & T_{xz}^{\text{diss}} &= -2\eta_{xy}u_{xz}, \\
 T_{yz}^{\text{diss}} &= -2\eta_{yz}u_{yz}.
 \end{aligned} \tag{A.4}$$

So far, we only considered the contribution to the stress tensor due to the dissipation via viscosity and found the terms in the constitutive equation which contain the velocity of the normal fluid u_μ . In general, also terms depending on the derivative of Nambu-Goldstone boson fields $v_\mu = \partial_\mu \varphi$, on the superfluid velocity and on the velocity of the director may contribute to the dissipative part of the stress tensor. Here the director is given by the vector pointing in the preferred direction. However these terms do not contribute to the off-diagonal components of the energy-momentum tensor for the following reasons: (1) a shear viscosity due to the superfluid velocity leads to a non-positive divergence of the entropy current [30, 31] and (2) no rank two tensor can be formed out of

degrees of freedom of the director if the gradients of the director vanish [15]. In our case the second argument is fulfilled since the condensate is homogenous and the fluctuations depend only on time. These degrees of freedom will generate additional transport coefficients, but they do not change the shear viscosities which we study in this letter. Thus we can write Kubo formulae which determine the shear viscosities in terms of the stress energy correlation functions as in (1).

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